

**Calculus I**  
**Chapter 5 Review**

1. Estimate  $\int_0^2 (x^3 + 5)dx$  using the given approximation method with  $n = 4$ .

a. LRAM

b. RRAM

c. TRAP

d. SIMP

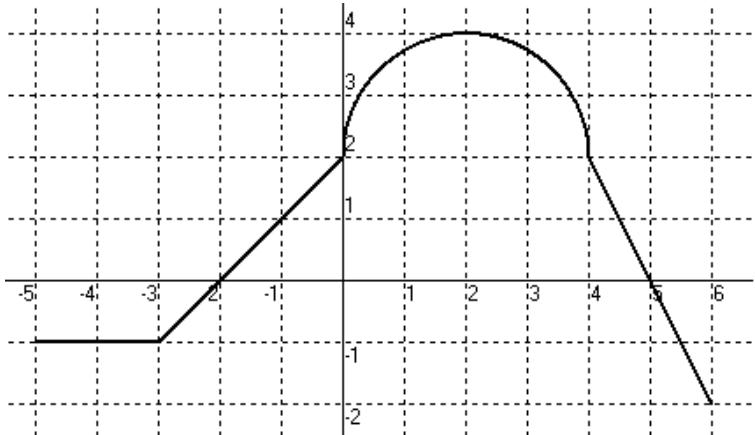
2. Evaluate each of the following given the graph of  $f(x)$ .

a.  $\int_{-5}^{-2} f(x)dx$

b.  $\int_{-5}^0 f(x)dx$

c.  $\int_0^4 3f(x)dx$

d.  $\int_6^0 f(x)dx$



3. If  $\int_0^3 g(x)dx = -6$ ,  $\int_0^8 g(x)dx = 15$ , and  $\int_0^3 h(x)dx = 3$ , find the following.

a.  $\int_0^3 \left( -2g(x) + \frac{1}{2}h(x) \right) dx$

b.  $\int_3^8 g(x)dx$

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4. Estimate  $\int_0^{10} f(x)dx$  from the data in the table using 6 intervals of *unequal* length and the given method.

$x$	0	2	3	5	8	9	10
$f(x)$	20	35	40	20	10	-30	-40

a. LRAM

b. RRAM

c. TRAP

d. SIMP

5. Evaluate each of the following using an area formula from geometry.

a.  $\int_{-3}^3 \sqrt{9-x^2} dx$

b.  $\int_0^3 |4-3x| dx$

6. Evaluate each of the following by setting up two separate integrals and integrating.

a.  $\int_0^3 |4-3x| dx$

b.  $\int_{-3}^3 f(x)dx$  if  $f(x)=\begin{cases} -x+3, & x \leq 0 \\ x^3+3, & x > 0 \end{cases}$

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**Evaluate the following.**

$$7. \int \sec 5x \tan 5x dx$$

$$8. \int \frac{\sin x dx}{\cos^3 x}$$

$$9. \int \frac{x^4 dx}{\sqrt{7x^5 + 2}}$$

$$10. \int \cot^9 x \csc^2 x dx$$

$$11. \int x \sqrt[5]{x-5} dx$$

$$12. \int (x-1)(x^2 - 2x)^5 dx$$

$$13. \int \sin 3x \cos 3x dx$$

$$14. \int x \cos 4x^2 dx$$

$$15. \int \sqrt{x}(x^2 + 2x) dx$$

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$$16. \int \csc^2 \pi x dx$$

$$17. \int_0^3 3x^2 + x - 2 dx$$

$$18. \int_0^2 \frac{x^2 dx}{\sqrt{x^3 + 1}}$$

$$19. \int_{\pi/4}^{3\pi/4} \csc^2 x dx$$

$$20. \int_0^\pi \sin \frac{x}{2} dx$$

**Find f(x) given the following information.**

$$21. f''(x) = x^2 - 3x + 2, \quad f'(2) = -1, \quad f(-2) = 1 \quad 22. f'(x) = 3 \sin\left(\frac{x}{4}\right), \quad f(0) = 2$$

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1. Estimate  $\int_0^2 (x^3 + 5)dx$  using the given approximation method with  $n = 4$ .

x	y
0	5
1/2	5.125
1	6
1.5	8.375
2	13

a. LRAM

$$\frac{1}{2}(5 + 5.125 + 6 + 8.375)$$

$$= \boxed{12.25}$$

b. RRAM

$$\frac{1}{2}(5.125 + 6 + 8.375 + 13)$$

$$= \boxed{16.25}$$

c. TRAP

$$\frac{1}{2}(\frac{1}{2})(5 + 2(5.125) + 2(6) + 2(8.375) + 13)$$

$$= \boxed{14.25}$$

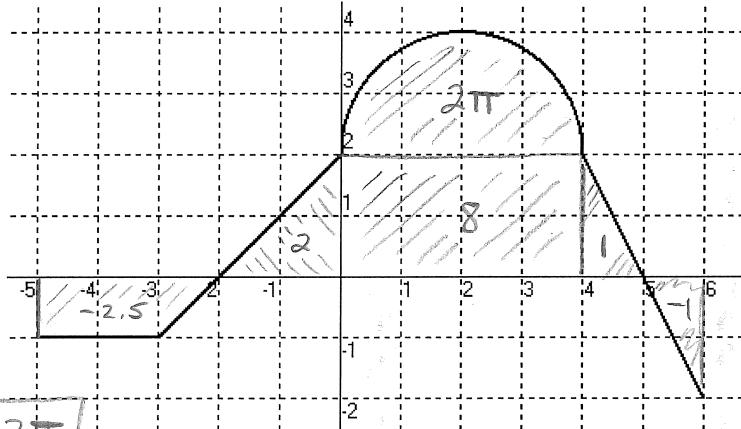
d. SIMP

$$\frac{1}{3}(\frac{1}{2})(5 + 4(5.125) + 2(6) + 4(8.375) + 13)$$

$$= \boxed{14}$$

2. Evaluate each of the following given the graph of  $f(x)$ .

a.  $\int_{-5}^{-2} f(x)dx = \boxed{-2.5}$



b.  $\int_{-5}^0 f(x)dx = -2.5 + 2 = \boxed{-1/2}$

c.  $\int_0^4 3f(x)dx = \boxed{3(8 + 2\pi)} = 24 + 6\pi$

d.  $\int_6^0 f(x)dx = - \int_0^6 f(x)dx = \boxed{-8 - 2\pi}$

3. If  $\int_0^3 g(x)dx = -6$ ,  $\int_0^8 g(x)dx = 15$ , and  $\int_0^3 h(x)dx = 3$ , find the following.

a.  $\int_0^3 \left(-2g(x) + \frac{1}{2}h(x)\right)dx$

$$-2 \int_0^3 g(x)dx + \frac{1}{2} \int_0^3 h(x)dx$$

$$= -2(-6) + \frac{1}{2}(3)$$

$$= \boxed{13.5}$$

b.  $\int_3^8 g(x)dx$

$$= \int_0^8 g(x)dx - \int_0^3 g(x)dx$$

$$= 15 - (-6)$$

$$= \boxed{21}$$

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4. Estimate  $\int_0^{10} f(x)dx$  from the data in the table using 6 intervals of *unequal* length and the given method.

$$\Delta x = \underbrace{2}_{\text{1st}}, \underbrace{1}_{\text{2nd}}, \underbrace{2}_{\text{3rd}}, \underbrace{3}_{\text{4th}}, \underbrace{1}_{\text{5th}}, \underbrace{1}_{\text{6th}}$$

x	0	2	3	5	8	9	10
f(x)	20	35	40	20	10	-30	-40

a. LRAM

$$2(20) + 1(35) + 2(40) + 3(20) + 1(10) \\ + 1(-30) \\ = \boxed{195}$$

b. RRAM

$$2(35) + 1(40) + 2(20) + 3(10) + 1(-30) \\ + 1(-40) \\ = \boxed{110}$$

c. TRAP

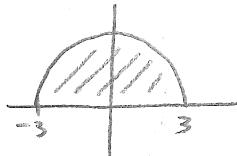
$$\frac{1}{2}(2)(20+35) + \frac{1}{2}(1)(35+40) + \frac{1}{2}(2)(40+20) \\ + \frac{1}{2}(3)(20+10) + \frac{1}{2}(1)(10-30) + \frac{1}{2}(1)(-30-40) \\ = \boxed{152.5}$$

d. SIMP

$$N/A$$

5. Evaluate each of the following using an area formula from geometry.

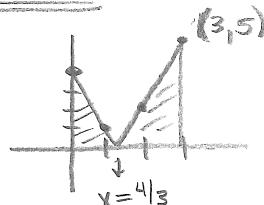
a.  $\int_{-3}^3 \sqrt{9-x^2} dx$



$$\frac{1}{2}\pi(3)^2$$

$$\boxed{\frac{9\pi}{2}}$$

b.  $\int_0^3 |4-3x| dx$



$$\frac{1}{2}(\frac{4}{3})(4) + \frac{1}{2}(\frac{2}{3})(5) \\ \frac{16}{6} + \frac{25}{6}$$

$$\boxed{41/6}$$

6. Evaluate each of the following by setting up two separate integrals and integrating.

a.  $\int_0^3 |4-3x| dx$  SKIP

b.  $\int_{-3}^3 f(x)dx$  if  $f(x) = \begin{cases} -x+3, & x \leq 0 \\ x^3+3, & x > 0 \end{cases}$

$$\int_{-3}^0 -x+3 dx + \int_0^3 x^3+3 dx \\ -\frac{1}{2}x^2+3x \Big|_{-3}^0 + \frac{1}{4}x^4+3x \Big|_0^3$$

$$\frac{1}{2}(-3)^2 - 3(-3) + \frac{1}{4}(3)^4 + 3(3)$$

$$13.5 + 29.25$$

$$\boxed{42.75}$$

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Evaluate the following.

7.  $\int \sec 5x \tan 5x dx$

$$\boxed{\frac{1}{5} \sec^2 5x + C}$$

$$u = 5x$$

$$\frac{du}{5} = \sec^2 x dx$$

$$\frac{1}{5} \int \sec u \tan u du$$

$$\frac{1}{5} \sec^2 u + C$$

10.  $\int \cot^9 x \csc^2 x dx$

$$u = \cot x$$

$$du = -\csc^2 x dx$$

$$-\int u^9 du$$

$$-\frac{1}{10} u^{10} + C$$

$$\boxed{-\frac{1}{10} \cot^{10} x + C}$$

13.  $\int \sin 3x \cos 3x dx$

$$u = \sin 3x$$

$$\frac{1}{3} \int u du$$

$$\frac{1}{6} u^2 + C$$

$$\boxed{\frac{1}{6} \sin^2 3x + C}$$

8.  $\int \frac{\sin x dx}{\cos^3 x}$

$$u = \cos x$$

$$du = -\sin x dx$$

$$-\int u^{-3} du$$

$$\frac{1}{2} u^{-2} + C$$

$$\boxed{\frac{1}{2} (\cos x)^{-2} + C}$$

11.  $\int x \sqrt[5]{x-5} dx$

$$u = x-5$$

$$du = dx$$

$$\int (u+5) u^{1/5} du$$

$$5u^{6/5} + 5u^{11/5} du$$

$$\frac{5}{11} u^{11/5} + \frac{25}{6} u^{6/5} + C$$

$$\boxed{\frac{5}{11} (x-5)^{11/5} + \frac{25}{6} (x-5)^{6/5} + C}$$

14.  $\int x \cos 4x^2 dx$

$$u = 4x^2$$

$$du = 8x dx$$

$$\frac{1}{8} \int \cos u du$$

$$\frac{1}{8} \sin u + C$$

$$\boxed{\frac{1}{8} \sin(4x^2) + C}$$

9.  $\int \frac{x^4 dx}{\sqrt{7x^5 + 2}}$

$$u = 7x^5 + 2$$

$$du = 35x^4 dx$$

$$\frac{1}{35} \int u^{-1/2} du$$

$$\frac{2}{35} u^{1/2} + C$$

$$\boxed{\frac{2}{35} (7x^5 + 2)^{1/2} + C}$$

12.  $\int (x-1)(x^2 - 2x)^5 dx$

$$u = x^2 - 2x$$

$$du = 2x-2 dx$$

$$2(x-1)$$

$$\frac{1}{2} \int u^5 du$$

$$\frac{1}{12} u^6 + C$$

$$\boxed{\frac{1}{12} (x^2 - 2x)^6 + C}$$

15.  $\int \sqrt{x}(x^2 + 2x) dx$

$$\int x^{5/2} + 2x^{3/2} dx$$

$$\boxed{\frac{2}{7} x^{7/2} + \frac{4}{5} x^{5/2} + C}$$

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16.  $\int \csc^2(\pi x) dx$

$$-\frac{1}{\pi} \cot(\pi x) + C$$

$$u = \pi x \\ du = \pi dx$$

$$\frac{1}{\pi} \int \csc^2 u du$$

$$-\frac{1}{\pi} \cot u + C$$

17.  $\int_0^3 3x^2 + x - 2 dx$

$$x^3 + \frac{1}{2}x^2 - 2x \Big|_0^3$$

$$3^3 + \frac{1}{2}(3)^2 - 2(3)$$

$$25.5 \\ \frac{57}{2}$$

18.  $\int_0^2 \frac{x^2 dx}{\sqrt{x^3 + 1}}$

$$\frac{1}{3} \int_1^9 u^{-1/2} du$$

$$\frac{2}{3} u^{1/2} \Big|_1^9$$

$$\frac{2}{3} (\sqrt{9} - \sqrt{1})$$

$$4 | 3$$

$$x=0 \Rightarrow u=1 \\ x=2 \Rightarrow u=9$$

19.  $\int_{\pi/4}^{3\pi/4} \csc^2 x dx$

$$-\cot x \Big|_{\pi/4}^{3\pi/4}$$

$$-\cot\left(\frac{3\pi}{4}\right) + \cot\left(\frac{\pi}{4}\right)$$

$$1 + 1$$

$$2$$

20.  $\int_0^{\pi} \sin \frac{x}{2} dx$

$$-2 \cos \frac{x}{2} \Big|_0^{\pi}$$

$$-2(\cos \frac{\pi}{2} - \cos 0)$$

$$-2(0 - 1)$$

$$2$$

$$u = x/2 \\ du = 1/2 dx$$

$$2 \int_0^{\pi/2} \sin u du$$

$$-2 \cos u \Big|_0^{\pi/2}$$

$$-2(\cos \frac{\pi}{2} - \cos 0)$$

$$-2(0 - 1)$$

$$2$$

$$x=0 \rightarrow u=0 \\ x=\pi \rightarrow u=\pi/2$$

Find  $f(x)$  given the following information.

21.  $f''(x) = x^2 - 3x + 2, \quad f'(2) = -1, \quad f(-2) = 1$

22.  $f'(x) = 3 \sin\left(\frac{x}{4}\right), \quad f(0) = 2$

$$f' = \frac{1}{3}x^3 - \frac{3}{2}x^2 + 2x + C$$

$$-1 = \frac{1}{3}(2)^3 - \frac{3}{2}(2)^2 + 2(2) + C$$

$$C = -\frac{5}{3}$$

$$\begin{cases} f' = \frac{1}{3}x^3 - \frac{3}{2}x^2 + 2x - \frac{5}{3} \end{cases}$$

$$f = \frac{1}{12}x^4 - \frac{1}{2}x^3 + x^2 - \frac{5}{3}x + C$$

$$1 = \frac{1}{12}(-2)^4 - \frac{1}{2}(-2)^3 + (-2)^2 - \frac{5}{3}(-2) + C \Rightarrow C = -\frac{35}{3}$$

$$f(x) = \frac{1}{12}x^4 - \frac{1}{2}x^3 + x^2 - \frac{5}{3}x - \frac{35}{3}$$

$$f = -12 \cos(\frac{x}{4}) + C$$

$$2 = -12 \cos(0) + C$$

$$14 = C$$

$$f = -12 \cos(\frac{x}{4}) + 14$$